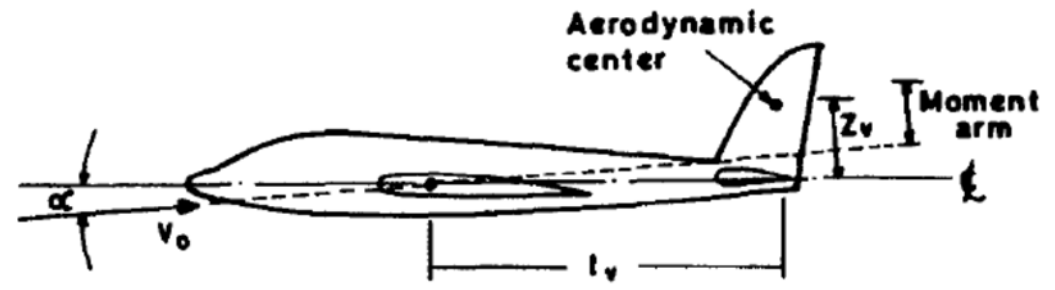


$$\begin{bmatrix} C_{y_\beta} & C_{y_{\delta_a}} & C_{y_{\delta_r}} \\ C_{l_\beta} & C_{l_{\delta_a}} & C_{l_{\delta_r}} \\ C_{n_\beta} & C_{n_{\delta_a}} & C_{n_{\delta_r}} \end{bmatrix} \begin{Bmatrix} \beta \\ \delta_a \\ \delta_r \end{Bmatrix} = \begin{Bmatrix} -\frac{(mg \sin \phi \cos \gamma + F_{y_{T_1}})}{\bar{q}_1 S} \\ -\frac{L_{T_1}}{\bar{q}_1 S b} \\ -\frac{N_{T_1} - \Delta N_{D_1}}{\bar{q}_1 S b} \end{Bmatrix}$$



a) Low angle of attack

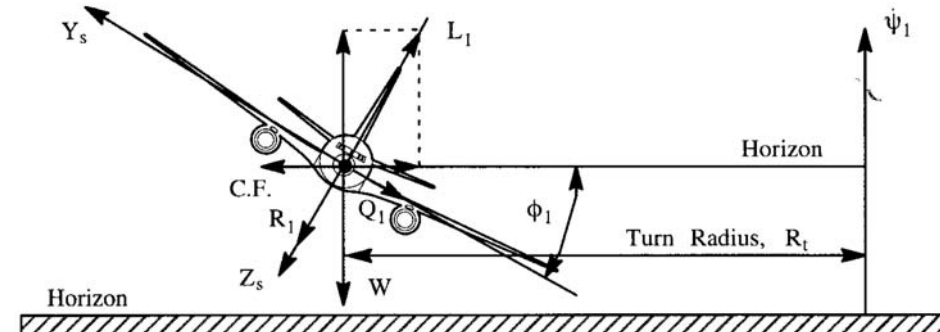
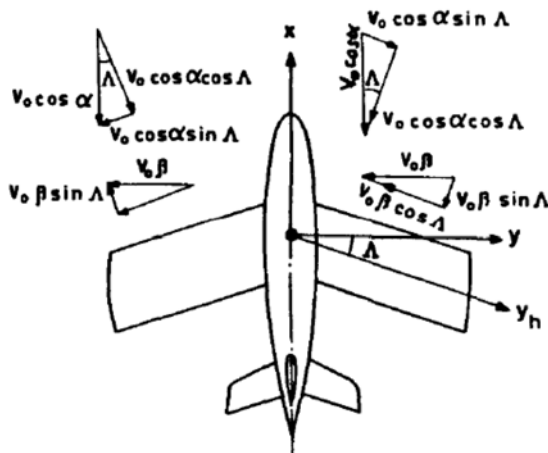
Estabilidad y Control Detallado

Equilibrado Lateral-Direccional

Tema 14.1

Sergio Esteban Roncero

Departamento de Ingeniería Aeroespacial
Y Mecánica de Fluidos



Equilibrado Lateral-Direccional - I

Vuelo rectilíneo y constante

$$\begin{aligned}
 -mg \sin \phi_1 \cos \gamma_1 &= (C_{y\beta} \beta_1 + C_{y\delta_a} \delta_{a_1} + C_{y\delta_r} \delta_{r_1}) \bar{q}_1 S + F_{y_{T_1}} \\
 0 &= (C_{l\beta} \beta_1 + C_{l\delta_a} \delta_{a_1} + C_{l\delta_r} \delta_{r_1}) \bar{q}_1 S b + L_{T_1} \\
 0 &= (C_{n\beta} \beta_1 + C_{n\delta_a} \delta_{a_1} + C_{n\delta_r} \delta_{r_1}) \bar{q}_1 S b + N_{T_1}
 \end{aligned}$$

Componente de empuje asimétrico

Sin asimetrías propulsivas, y con la línea de empuje neto pasa por el Xcg

$$L_{T_1} = N_{T_1} = F_{T_{y_1}} = 0.$$

Fallo de motor crea aumento de resistencia  Momento de guiñada adicional

$$N_{T_1} + \Delta N_{D_1} \approx (F_{OEI}) N_{T_1}$$

Table 4.2 Effect of the Propulsive Installation on F_{OEI} Eqn (4.72)				
Type of Powerplant	Fixed Pitch	Variable Pitch	Low BPR	High BPR
F_{OEI}	1.25	1.10	1.15	1.25

Equilibrado Lateral-Direccional - II

Thrust induced rolling moment

Thrust induced side force

Thrust induced yawing moment

$$\left. \begin{array}{l} \longrightarrow \left\{ L_T \right\} \\ \longrightarrow \left\{ F_{T_y} \right\} \\ \longrightarrow \left\{ N_T \right\} \end{array} \right\} \tau = \left. \begin{array}{l} \left\{ \sum_{i=0}^{i=n} T_i (z_{T_i} \psi_{T_i} - y_{T_i} \phi_{T_i}) - T_i y_{T_i} \alpha_1 \right\} \\ \left\{ \sum_{i=0}^{i=n} T_i \psi_{T_i} \right\} \\ \left\{ \sum_{i=0}^{i=n} T_i (x_{T_i} \psi_{T_i} - y_{T_i} \phi_{T_i}) + \Delta N_{D_i} \right\} \end{array} \right\}$$

Momentos = 0
Configuración simétrica

$$L_{T_{ix}} = L_T = \left[\sum_{i=0}^{i=n} T_i (-z_{T_i} \cos \phi_{T_i} \sin \psi_{T_i} - y_{T_i} \sin \phi_{T_i}) \right] \cos \alpha_1 + \tau \left[\sum_{i=0}^{i=n} T_i (x_{T_i} \cos \phi_{T_i} \sin \psi_{T_i} - y_{T_i} \cos \phi_{T_i} \cos \psi_{T_i}) \right] \sin \alpha_1 \quad (3.92a)$$

$$F_{T_{iy}} = F_{T_y} = \sum_{i=0}^{i=n} T_i (\cos \phi_{T_i} \sin \psi_{T_i}) \quad (3.92b)$$

$$N_{T_{ix}} = N_T = \left[\sum_{i=0}^{i=n} T_i (x_{T_i} \cos \phi_{T_i} \sin \psi_{T_i} - y_{T_i} \cos \phi_{T_i} \cos \psi_{T_i}) \right] \cos \alpha_1 + \left[\sum_{i=0}^{i=n} T_i (-z_{T_i} \cos \phi_{T_i} \sin \psi_{T_i} - y_{T_i} \sin \phi_{T_i}) \right] \sin \alpha_1 \quad (3.92c)$$

Estudio Completo → Necesario realizar simplificaciones

Equilibrado Lateral-Direccional - III

Configuración OEI - Asimétrico

$$L_T = [T_i(z_{T_i} \cos \phi_{T_i} \sin \psi_{T_i} - y_{T_i} \sin \phi_{T_i})] \cos \alpha_1 + [T_i(x_{T_i} \cos \phi_{T_i} \sin \psi_{T_i} - y_{T_i} \cos \phi_{T_i} \cos \psi_{T_i})] \sin \alpha_1$$

$$F_{T_y} = T_i(\cos \phi_{T_i} \sin \psi_{T_i})$$

$$N_T = [T_i(x_{T_i} \cos \phi_{T_i} \sin \psi_{T_i} - y_{T_i} \cos \phi_{T_i} \cos \psi_{T_i})] \cos \alpha_1 - [T_i(z_{T_i} \cos \phi_{T_i} \sin \psi_{T_i} - y_{T_i} \sin \phi_{T_i})] \sin \alpha_1 + \Delta N_{D_i}$$

the thrust-line inclination angle ϕ_{T_i}

lateral thrust-line off-set angle, ψ_{T_i}

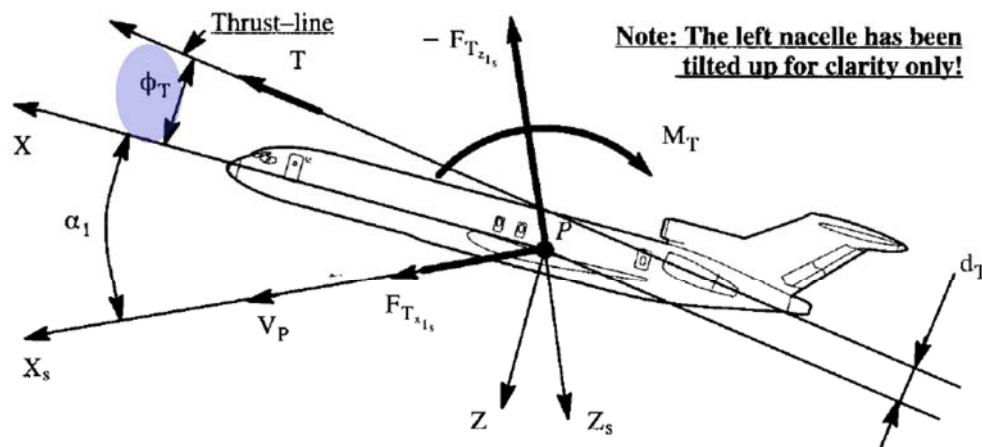


Figure 3.26 Steady State Thrust Forces and Pitching Moment in Stability Axes

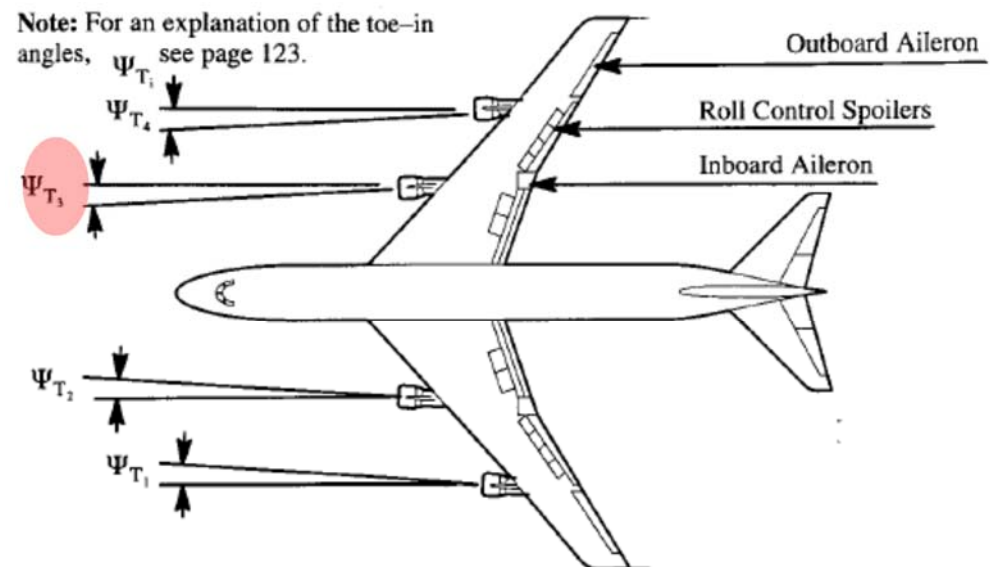
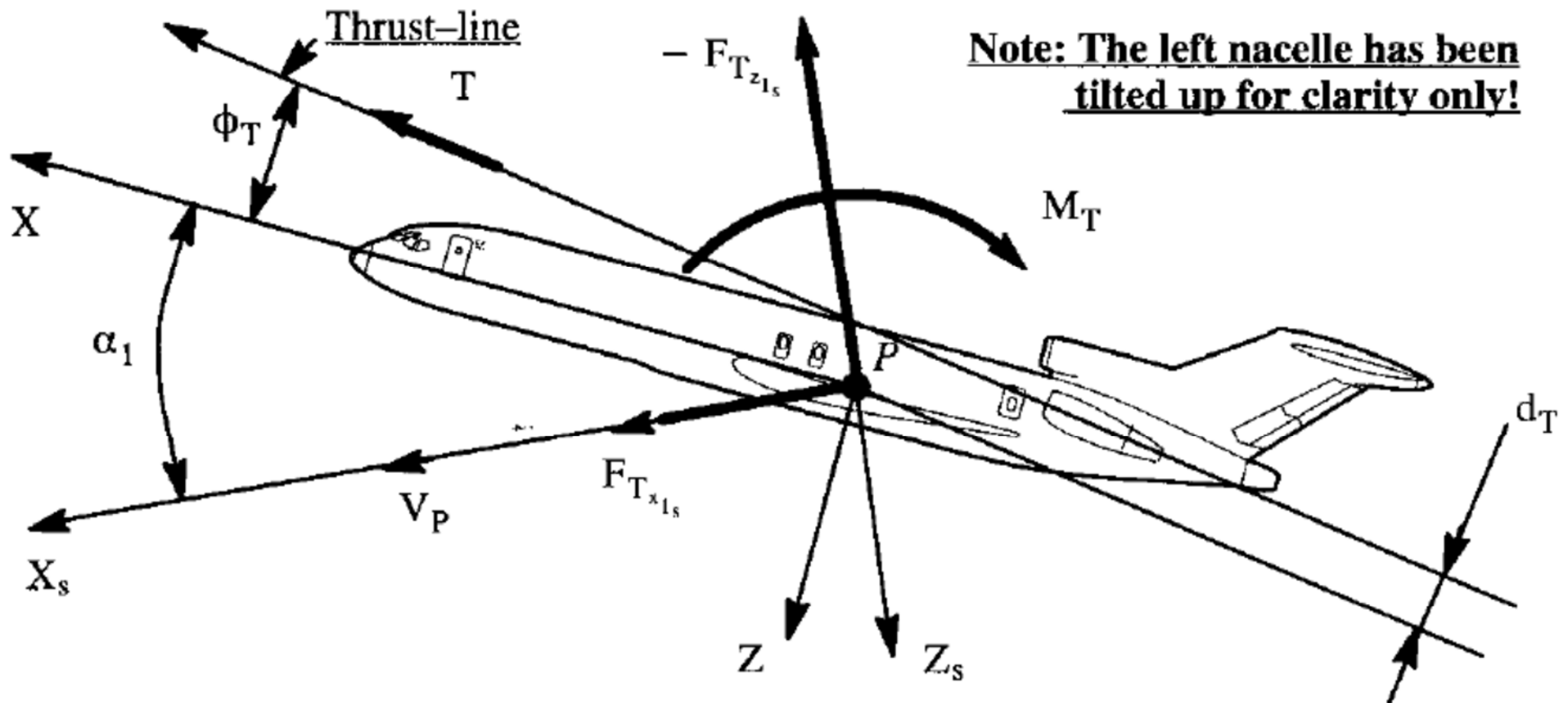


Figure 3.36 Boeing Model 747 with Three Types of Lateral Control

Equilibrado Lateral-Direccional - IV



Note: The left nacelle has been tilted up for clarity only!

the thrust-line inclination angle ϕ_{T_i}

Figure 3.26 Steady State Thrust Forces and Pitching Moment in Stability Axes

Equilibrado Lateral-Direccional - V

Note: For an explanation of the toe-in angles, ψ_{T_i} see page 123.

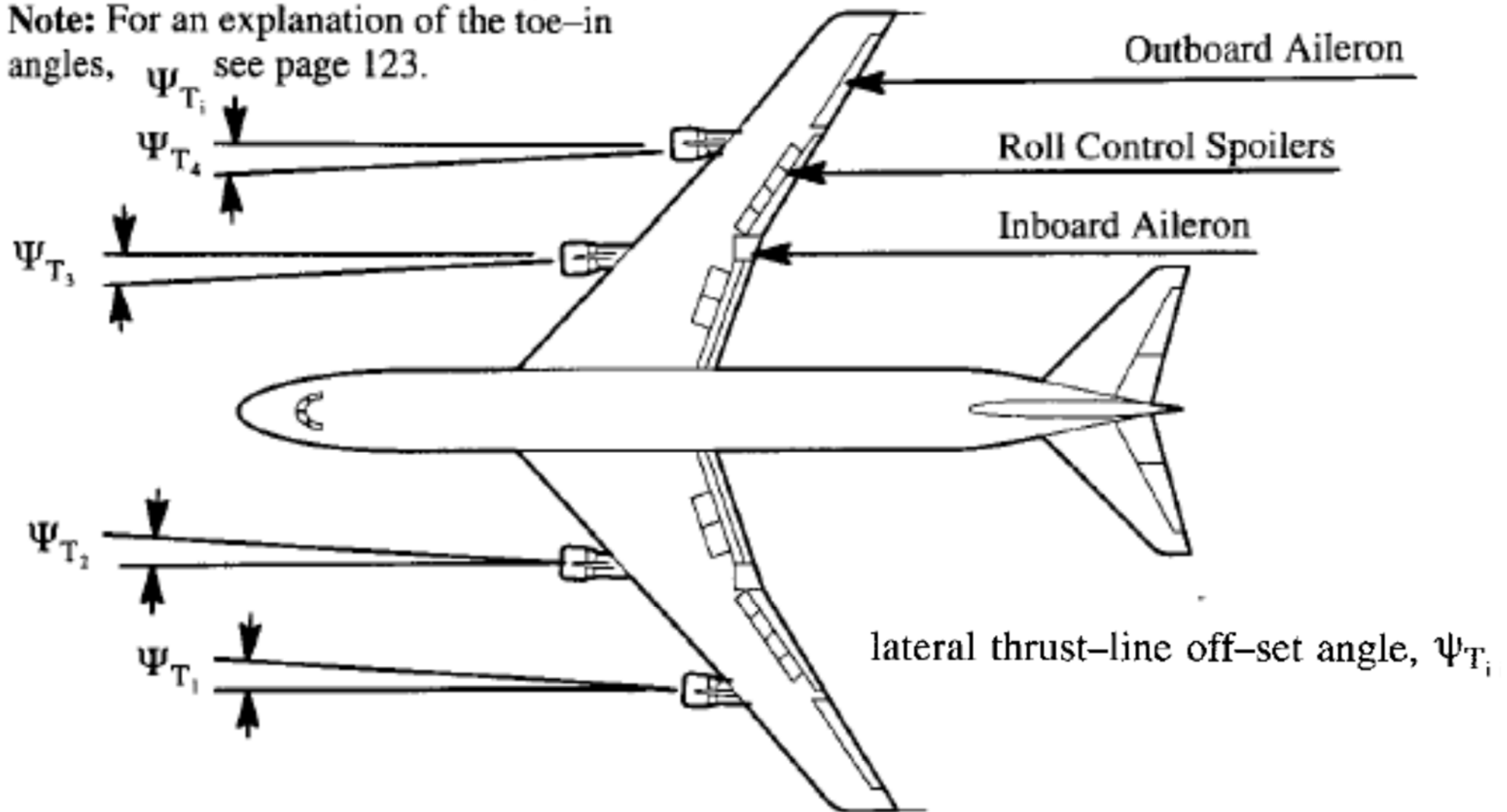


Figure 3.36 Boeing Model 747 with Three Types of Lateral Control

Equilibrado Lateral-Direccional - VI

Si se asume que el ángulo de ataque de equilibrio, y los ángulos de toe-up y toe-down son pequeños

$$L_T \approx T_i(z_{T_i}\psi_{T_i} - y_{T_i}\phi_{T_i}) - T_i y_{T_i} \alpha_1$$

$$N_T \approx T_i(x_{T_i}\psi_{T_i} - y_{T_i}) + \Delta N_{D_i}$$

$$F_{T_y} = T_i \psi_{T_i}$$

$$T_{i_x} = T_i \cos \phi_{T_i} \cos \psi_{T_i}$$

$$T_{i_y} = T_i \cos \phi_{T_i} \sin \psi_{T_i}$$

$$T_{i_z} = T_i \sin \phi_{T_i}$$

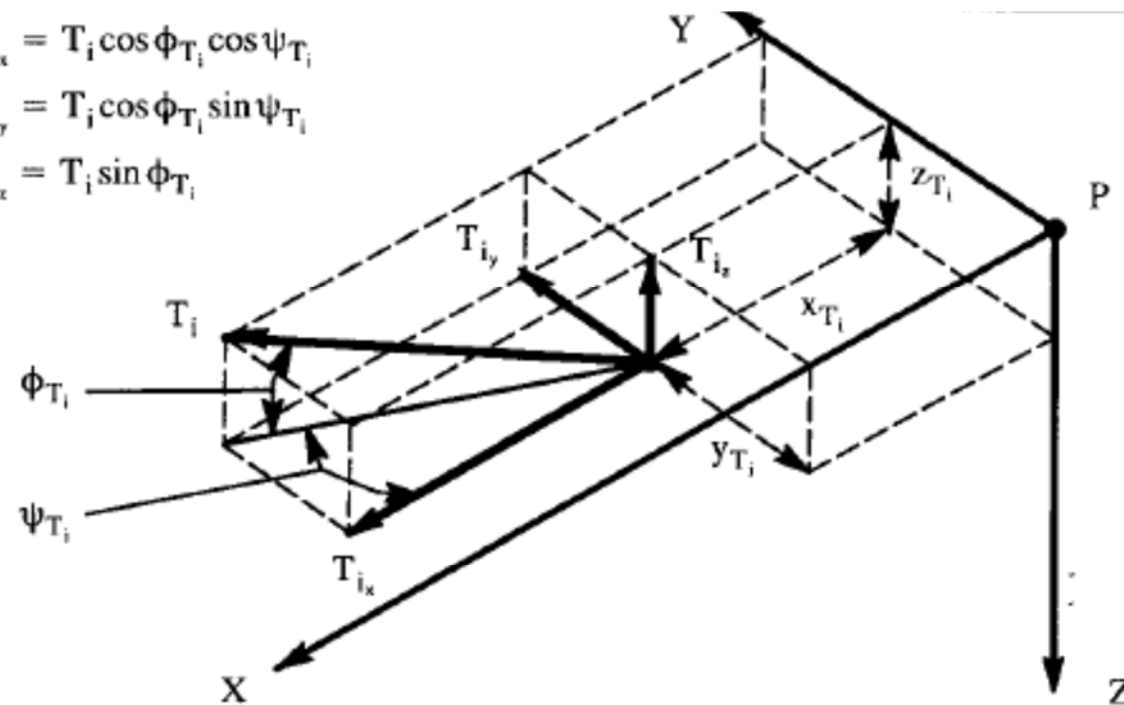


Figure 3.25 Location of Engine Thrust-line and Point of Thrust Application

Equilibrado Lateral-Direccional - VII

Para un empuje (T) y ángulo de planeo γ $\gamma = \sin^{-1}\left(\frac{T-D}{W}\right)$

ϕ , β , δ_a and δ_r

select: ϕ and solve for β , δ_a and δ_r

select: β and solve for ϕ , δ_a and δ_r

select: δ_a and solve for ϕ , β and δ_r

select: δ_r and solve for ϕ , β and δ_a

$$\begin{bmatrix} C_{y_\beta} & C_{y_{\delta_a}} & C_{y_{\delta_r}} \\ C_{l_\beta} & C_{l_{\delta_a}} & C_{l_{\delta_r}} \\ C_{n_\beta} & C_{n_{\delta_a}} & C_{n_{\delta_r}} \end{bmatrix} \begin{Bmatrix} \beta \\ \delta_a \\ \delta_r \end{Bmatrix} = \begin{Bmatrix} \frac{-(mg \sin \phi \cos \gamma + F_{y_{T_1}})}{\bar{q}_1 S} \\ \frac{-L_{T_1}}{\bar{q}_1 S b} \\ \frac{-N_{T_1} - \Delta N_{D_1}}{\bar{q}_1 S b} \end{Bmatrix}$$

- Fallo de motor (One Engine Inoperative OEI)
 - Avión tiene que ser controlable en línea recta.
 - Ángulo de balance $\phi < 5^\circ$ para $V > 1.2 V_{stall}$
 - Se tiene que mantener el flujo de la corriente pegado δ_a o $\delta_r < 25^\circ$ (20° como max)

F_y Thrust induced side force $F_y \sim 0$

L_y Thrust induced rolling moment $L_y \sim 0$

N_y Thrust induced yawing moment $N_y \sim N_{T_1} + \Delta N_{D_1}$

Equilibrado Lateral-Direccional - III

$$\beta_1 = \frac{\begin{vmatrix} \frac{-(mg \sin \phi \cos \gamma + F_{y_{T_1}})}{\bar{q}_1 S} & C_{y_{\delta_a}} & C_{y_{\delta_r}} \\ \frac{-L_{T_1}}{\bar{q}_1 S b} & C_{l_{\delta_a}} & C_{l_{\delta_r}} \\ \frac{-N_{T_1} - \Delta N_{D_1}}{\bar{q}_1 S b} & C_{n_{\delta_a}} & C_{n_{\delta_r}} \end{vmatrix}}{\Delta}$$

$$\Delta = \begin{vmatrix} C_{y_{\beta}} & C_{y_{\delta_a}} & C_{y_{\delta_r}} \\ C_{l_{\beta}} & C_{l_{\delta_a}} & C_{l_{\delta_r}} \\ C_{n_{\beta}} & C_{n_{\delta_a}} & C_{n_{\delta_r}} \end{vmatrix}$$

$$\delta_{a_1} = \frac{\begin{vmatrix} C_{y_{\beta}} & \frac{-(mg \sin \phi \cos \gamma + F_{y_{T_1}})}{\bar{q}_1 S} & C_{y_{\delta_r}} \\ C_{l_{\beta}} & \frac{-L_{T_1}}{\bar{q}_1 S b} & C_{l_{\delta_r}} \\ C_{n_{\beta}} & \frac{-N_{T_1} - \Delta N_{D_1}}{\bar{q}_1 S b} & C_{n_{\delta_r}} \end{vmatrix}}{\Delta}$$

$$\delta_{r_1} = \frac{\begin{vmatrix} C_{y_{\beta}} & C_{y_{\delta_a}} & \frac{-(mg \sin \phi \cos \gamma + F_{y_{T_1}})}{\bar{q}_1 S} \\ C_{l_{\beta}} & C_{l_{\delta_a}} & \frac{-L_{T_1}}{\bar{q}_1 S b} \\ C_{n_{\beta}} & C_{n_{\delta_a}} & \frac{-N_{T_1} - \Delta N_{D_1}}{\bar{q}_1 S b} \end{vmatrix}}{\Delta}$$

Deflexiones Timón de Cola – OEI - I

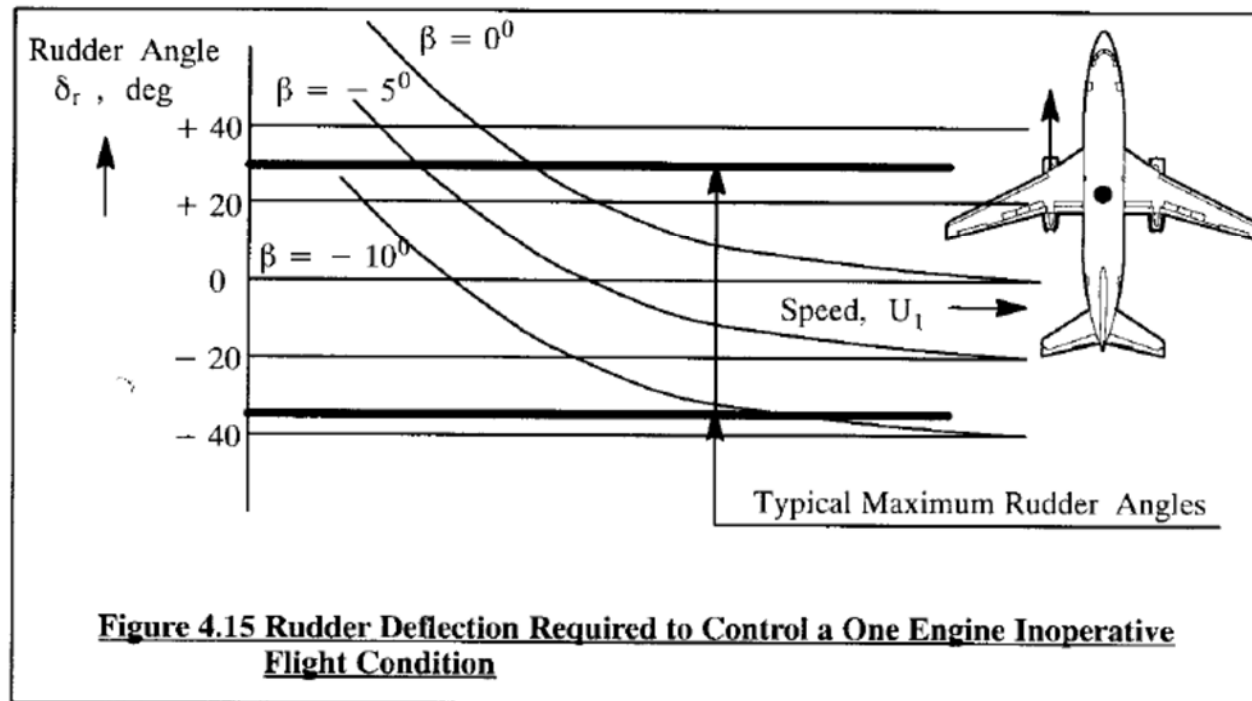
Análisis Simplificado

$$0 = (C_{n_\beta} \beta_1 + C_{n_{\delta_a}} \delta_{a_1} + C_{n_{\delta_r}} \delta_{r_1}) \bar{q}_1 S b + N_{T_1} \rightarrow C_{n_\beta} \beta + C_{n_{\delta_r}} \delta_r + \frac{N_{T_1} + \Delta N_{D_1}}{\bar{q}_1 S b} = 0$$

Cálculo de la cantidad de timón de dirección requerido para condición OEI

$$\delta_r = \frac{-C_{n_\beta} \beta - \frac{N_{T_1} + \Delta N_{D_1}}{\bar{q}_1 S b}}{C_{n_{\delta_r}}}$$

Estudio de sensibilidad: variación β, U_1, δ_r



$V > 1.2 V_{stall}$

Figure 4.15 Rudder Deflection Required to Control a One Engine Inoperative Flight Condition

Deflexiones Timón de Cola – OEI - II

Es deseable volar con $\beta \approx 0^\circ$ para reducir resistencia

$$\delta_r = \frac{-C_{n_\beta} \beta - \frac{N_{T_1} + \Delta N_{D_1}}{\bar{q}_1 S b}}{C_{n_{\delta_r}}} \quad \longrightarrow \quad \delta_r = - \left[\frac{N_{T_1} + \Delta N_{D_1}}{C_{n_{\delta_r}} \bar{q}_1 S b} \right]$$

Para deflexiones superiores a 25° el timón de dirección puede entrar en pérdida

Fijando $\delta_{r_{max}}$

$$\delta_{r_{max}} \quad \longrightarrow \quad V_{mc} = \sqrt{\frac{2(N_{T_1} + \Delta N_{D_1})}{\rho C_{n_{\delta_r}} \delta_{r_{max}} S b}}$$

V_{mc} es la mínima velocidad a la que puede ser controlado el avión en condición OEI

$$V_{mc} \leq 1.2V_{s_{OEI}} \text{ (FAR 23 and FAR 25)}$$

$$V_{mc} \leq \text{highest of } 1.1V_s \text{ or } V_s + 10 \text{ keas (Mil - F - 8785C)}$$

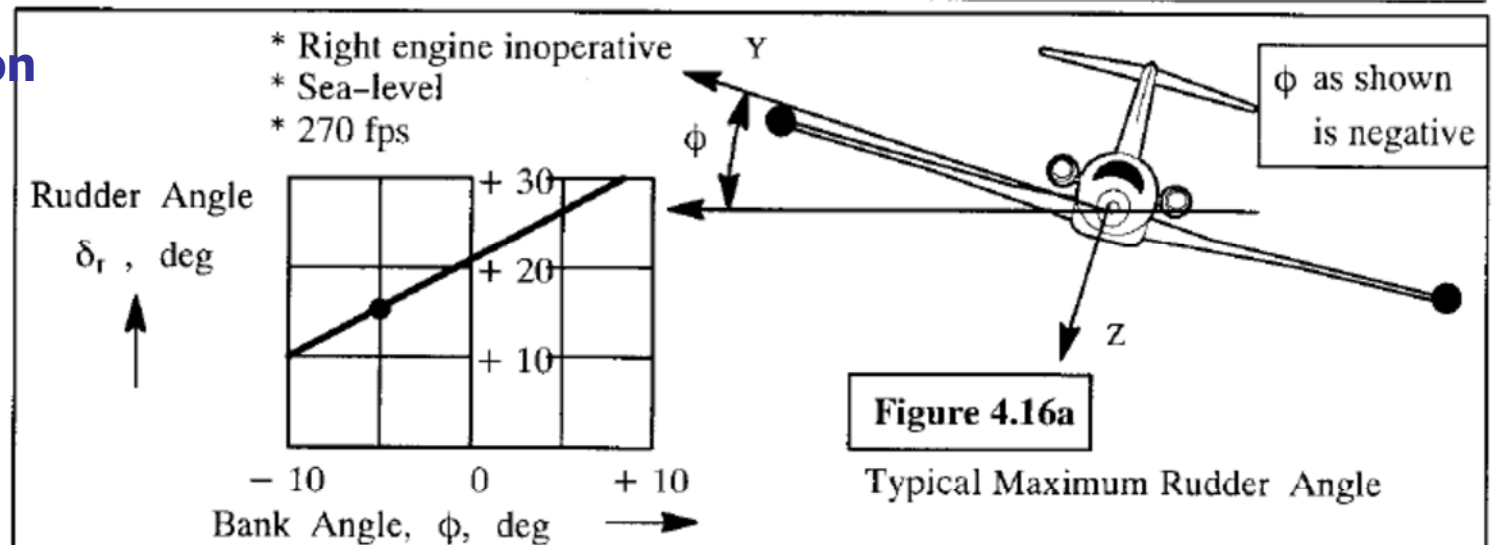
Potencia de control del timón de dirección

$$C_{n_{\delta_r}}$$

Deflexiones Timón de Cola – OEI - III

Cantidad de Timón de dirección se puede reducir si se permite $\phi > 0$

Variación timón dirección Vs Ángulo de balance



Variación timón dirección Vs. Tamaño Deriva Vertical

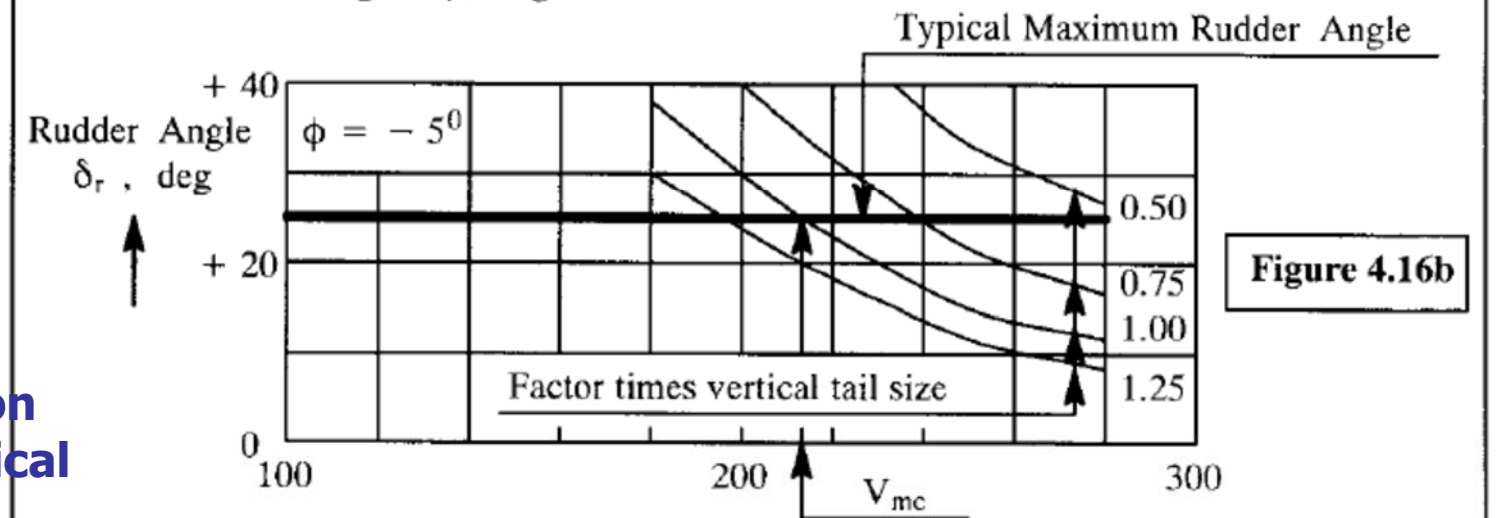


Figure 4.16 Effect of Bank Angle, Vertical Tail Size and Airspeed on the Rudder Angle Required to Hold a One Engine Inoperative Flight Condition

Deflexiones Alerón – OEI - I

- Después de un fallo de motor, previo a la acción del piloto se produce un deslizamiento

$$\beta_{\max} = - \left(\frac{N_{T_1} + \Delta N_{D_1}}{C_{n\beta} \bar{q}_1 S b} \right)$$

Análisis Simplificado

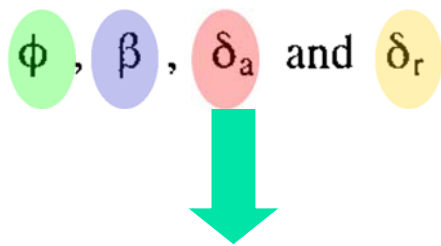
Para mantener las alas niveladas la cantidad de deflexión de alerón

$$\delta_a = \frac{- C_{l\beta} \beta_{\max} - \frac{L_{T_1}}{\bar{q}_1 S b}}{C_{l\delta_a}} = \frac{\left\{ \frac{C_{l\beta}}{C_{n\beta}} (N_{T_1} + \Delta N_{D_1}) - L_{T_1} \right\}}{C_{l\delta_a} \bar{q}_1 S b}$$

Para deflexiones superiores a 25° el alerón puede entrar en pérdida (20° como max)

Estudio Equilibrio Lateral-Direccional

- Cuando no hay fallo de motor (aviones monomotores) Se determinará a partir de fijar el ángulo de deslizamiento de $\beta \approx 15^\circ$
- Determinar la cantidad de deflexión de alerón y timón de dirección necesaria para equilibrar:
 - no tiene que ser superior al ángulo de deflexión para el que el ala entra en pérdida: Aproximadamente no superior a 25° (20° max)



Para un empuje (T) y ángulo de planeo γ

$$\gamma = \sin^{-1} \left(\frac{T - D}{W} \right)$$

select: ϕ and solve for β , δ_a and δ_r

select: β and solve for ϕ , δ_a and δ_r

select: δ_a and solve for ϕ , β and δ_r

select: δ_r and solve for ϕ , β and δ_a

$$\begin{bmatrix} C_{y_\beta} & C_{y_{\delta_a}} & C_{y_{\delta_r}} \\ C_{l_\beta} & C_{l_{\delta_a}} & C_{l_{\delta_r}} \\ C_{n_\beta} & C_{n_{\delta_a}} & C_{n_{\delta_r}} \end{bmatrix} \begin{Bmatrix} \beta \\ \delta_a \\ \delta_r \end{Bmatrix}$$

$$= \begin{Bmatrix} \frac{-(mg \sin \phi \cos \gamma + F_{y_{T_1}})}{\bar{q}_1 S} \\ \frac{-L_{T_1}}{\bar{q}_1 S b} \\ \frac{-N_{T_1} - \Delta N_{D_1}}{\bar{q}_1 S b} \end{Bmatrix}$$

F_y Thrust induced side force $F_y \sim 0$

L_y Thrust induced rolling moment $L_y \sim 0$

N_y Thrust induced yawing moment $N_y \sim 0$

Análisis de Vuelo Equilibrado

ONE ENGINE INOPERATIVE AND SIDESLIP ANGLE ANALYSIS

● Ecuaciones para el estudio de trimado lateral-direccional

● Características de las condiciones de fallo de motor

- El usuario introduce las condiciones de vuelo para el caso de fallo de motor:
 - Altitud, ángulo de resbalamiento, relación e la velocidad normalizada con la velocidad de entrada en pérdida, y relación de la potencia con respecto a la potencia máxima

● Matriz de los resultados

- El usuario puede modificar los valores (por ejemplo ampliando la potencia de control del alerón $C_{L\delta a}$ o la potencia de control del timón de dirección $C_{N\delta r}$)
- Sólo representa si es un vector de al menos 2 elementos

● Características de las condiciones de resbalamiento

El usuario introduce las condiciones de vuelo para el caso de resbalamiento:

- Altitud, ángulo de resbalamiento, relación e la velocidad normalizada con la velocidad de entrada en pérdida

ONE ENGINE INOPERATIVE AND SIDESLIP ANGLE ANALYSIS

$$\begin{bmatrix} C_{Y\beta} & C_{Y\delta a} & C_{Y\delta r} \\ C_{l\beta} & C_{l\delta a} & C_{l\delta r} \\ C_{n\beta} & C_{n\delta a} & C_{n\delta r} \end{bmatrix} \begin{bmatrix} \beta \\ \delta a \\ \delta r \end{bmatrix} = \begin{bmatrix} \frac{-(mg \sin(\phi) \cos(\gamma) + F_{YT_1})}{q S_{ref}} \\ \frac{-L_{T_1}}{q S_{ref} b} \\ \frac{-(N_{T_1} + \Delta N_{D_1})}{q S_{ref} b} \end{bmatrix}$$

MATRIX

-0.58038	0	0.17806
0.089531	0.1417	0.033163
0.11909	-0.059631	-0.057895

RESTORE INITIAL VALUES

ONE ENGINE INOPERATIVE

h (ft) 0 Estudio de condiciones

beta (°) 11

V/Vstall 1.2

P/Pmax 0.5

SOLVE

phi -28.811 da -61.2097 dr 231.844

SIDESLIP ANGLE

h (ft) 0 Estudio de condiciones

beta (°) 11

V/Vstall 1.2

SOLVE

phi -0.4775 da -16.1343 dr 39.2427

EDIT MODEL SAVE CHANGES CLOSE

● Resultados del trimado lateral-direccional

El usuario puede visualizar los resultados del trimado lateral direccional para tanto el caso de fallo de motor y ángulo de resbalamiento

Viraje Estacionario - I

$$0 = -(C_{D_0} + C_{D_\alpha} \alpha_1 + C_{D_{i_h}} i_{h_1} + C_{D_{\delta_e}} \delta_{e_1}) \bar{q}_1 S + T_1 \cos(\phi_T + \alpha_1)$$

$$mU_1 R_1 - mg \sin \phi_1 = (C_{y_\beta} \beta_1 + C_{Y_r} \frac{R_1 b}{2U_1} + C_{y_{\delta_a}} \delta_{a_1} + C_{y_{\delta_r}} \delta_{r_1}) \bar{q}_1 S$$

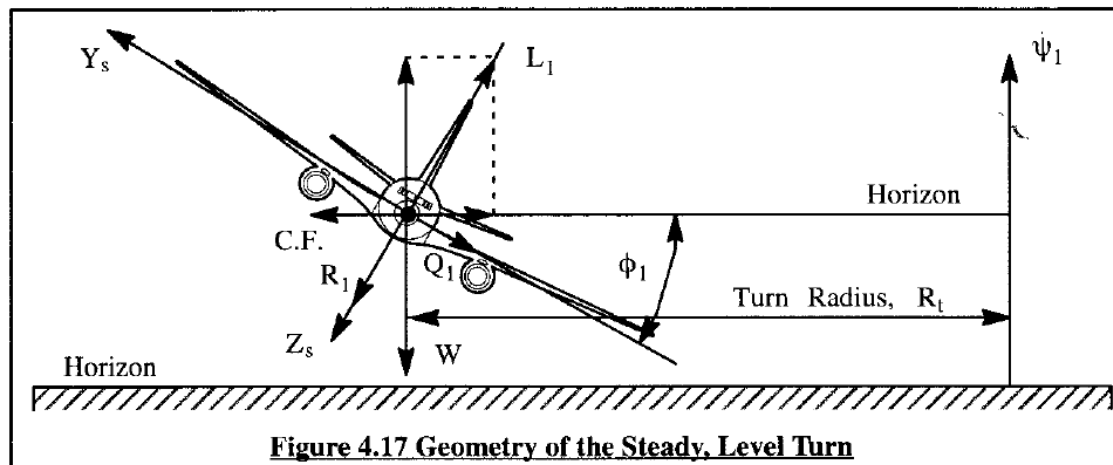
$$-mU_1 Q_1 - mg \cos \phi_1 = -(C_{L_0} + C_{L_\alpha} \alpha_1 + C_{L_q} \frac{Q_1 \bar{c}}{2U_1} + C_{L_{i_h}} i_{h_1} + C_{L_{\delta_e}} \delta_{e_1}) \bar{q}_1 S - T_1 \sin(\phi_T + \alpha_1)$$

$$(I_{zz} - I_{yy}) R_1 Q_1 = (C_{l_\beta} \beta_1 + C_{l_r} \frac{R_1 b}{2U_1} + C_{l_{\delta_a}} \delta_{a_1} + C_{l_{\delta_r}} \delta_{r_1}) \bar{q}_1 S b$$

$$-I_{xz} R_1^2 = (C_{m_0} + C_{m_\alpha} \alpha_1 + C_{m_q} \frac{Q_1 \bar{c}}{2U_1} + C_{m_{i_h}} i_{h_1} + C_{m_{\delta_e}} \delta_{e_1}) \bar{q}_1 S \bar{c}$$

$$I_{xz} Q_1 R_1 = (C_{n_\beta} \beta_1 + C_{n_r} \frac{R_1 b}{2U_1} + C_{n_{\delta_a}} \delta_{a_1} + C_{n_{\delta_r}} \delta_{r_1}) \bar{q}_1 S b$$

Sin asimetrías propulsivas, y con la línea de empuje neto pasa por el Xcg



$$P_1 = 0$$

$$Q_1 = \dot{\psi}_1 \sin \phi_1$$

$$R_1 = \dot{\psi}_1 \cos \phi_1$$

$$M_{T_1} = L_{T_1} = N_{T_1} = F_{T_{y_1}} = 0 ..$$

Viraje Estacionario - II

Condiciones de equilibrio en Viraje Estacionario

Turn radius

$$W = L \cos \phi_1 \quad \Rightarrow \quad U_1 = R_t \psi_1 \quad \Rightarrow \quad R_t = \frac{U_1^2}{g \tan \phi_1}$$

$$Q_1 = \frac{g \sin^2 \phi_1}{U_1 \cos \phi_1} = \frac{g}{U_1} \left(n - \frac{1}{n} \right) \quad \psi_1 = \frac{g \tan \phi_1}{U_1} \quad \leftarrow \quad n = 1/\cos \phi_1$$

and

$$R_1 = \frac{g \sin \phi_1}{U_1} = \frac{g}{n U_1} \sqrt{n^2 - 1}$$

Turn rate

$$L = nW$$

Factor de carga

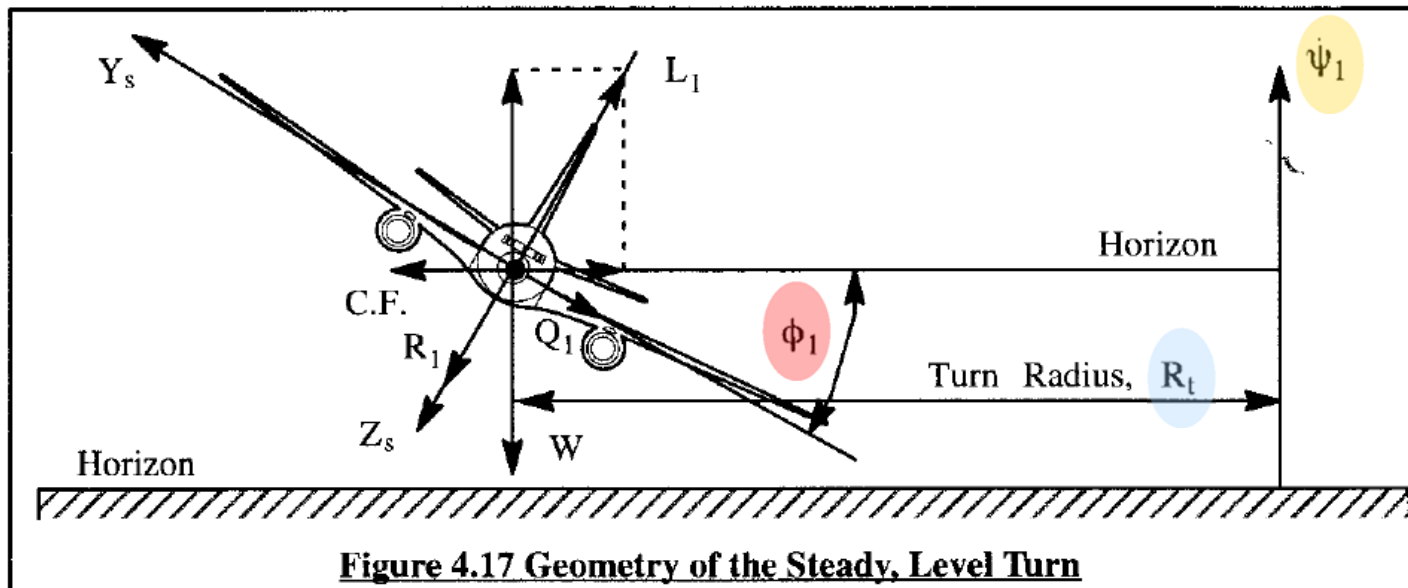


Figure 4.17 Geometry of the Steady, Level Turn

Viraje Estacionario - III

$$mU_1 R_1 - mg \sin \phi_1 = (C_{y_\beta} \beta_1 + C_{Y_r} \frac{R_1 b}{2U_1} + C_{y_{\delta_a}} \delta_{a_1} + C_{y_{\delta_r}} \delta_{r_1}) \bar{q}_1 S$$

$$(I_{zz} - I_{yy}) R_1 Q_1 = (C_{l_\beta} \beta_1 + C_{l_r} \frac{R_1 b}{2U_1} + C_{l_{\delta_a}} \delta_{a_1} + C_{l_{\delta_r}} \delta_{r_1}) \bar{q}_1 S b$$

$$I_{xz} Q_1 R_1 = (C_{n_\beta} \beta_1 + C_{n_r} \frac{R_1 b}{2U_1} + C_{n_{\delta_a}} \delta_{a_1} + C_{n_{\delta_r}} \delta_{r_1}) \bar{q}_1 S b$$

Lateral directional-equations

$$\begin{bmatrix} C_{y_\beta} & C_{y_{\delta_a}} & C_{y_{\delta_r}} \\ C_{l_\beta} & C_{l_{\delta_a}} & C_{l_{\delta_r}} \\ C_{n_\beta} & C_{n_{\delta_a}} & C_{n_{\delta_r}} \end{bmatrix} \begin{Bmatrix} \beta \\ \delta_a \\ \delta_r \end{Bmatrix} = \begin{Bmatrix} - C_{y_r} \frac{b g \sin \phi}{2U_1^2} \\ \frac{(I_{zz} - I_{yy}) g^2 \sin^3 \phi}{\bar{q}_1 S b U_1^2 \cos \phi} - C_{l_r} \frac{b g \sin \phi}{2U_1^2} \\ \frac{I_{xz} g^2 \sin^3 \phi}{\bar{q}_1 S b U_1^2 \cos \phi} - C_{n_r} \frac{b g \sin \phi}{2U_1^2} \end{Bmatrix}$$

Viraje Estacionario - IV

$$\beta_1 = \frac{\begin{vmatrix} a_{11} & C_{y\delta_a} & C_{y\delta_r} \\ b_{11} & C_{l\delta_a} & C_{l\delta_r} \\ c_{11} & C_{n\delta_a} & C_{n\delta_r} \end{vmatrix}}{\Delta}$$

where: $\Delta = \begin{vmatrix} C_{y\beta} & C_{y\delta_a} & C_{y\delta_r} \\ C_{l\beta} & C_{l\delta_a} & C_{l\delta_r} \\ C_{n\beta} & C_{n\delta_a} & C_{n\delta_r} \end{vmatrix}$

and: $a_{11} = -C_{y_r} \frac{bg \sin \phi}{2U_1^2}$

$$\delta_{a_1} = \frac{\begin{vmatrix} C_{y\beta} & a_{11} & C_{y\delta_r} \\ C_{l\beta} & b_{11} & C_{l\delta_r} \\ C_{n\beta} & c_{11} & C_{n\delta_r} \end{vmatrix}}{\Delta}$$

$$b_{11} = \frac{(I_{zz} - I_{yy})g^2 \sin^3 \phi}{\bar{q}_1 S b U_1^2 \cos \phi} - C_{l_r} \frac{g b \sin \phi}{2U_1^2}$$

$$c_{11} = \frac{I_{xz} g^2 \sin^3 \phi}{\bar{q}_1 S b U_1^2 \cos \phi} - C_{n_r} \frac{g b \sin \phi}{2U_1^2}$$

$$\delta_{r_1} = \frac{\begin{vmatrix} C_{y\beta} & C_{y\delta_a} & a_{11} \\ C_{l\beta} & C_{l\delta_a} & b_{11} \\ C_{n\beta} & C_{n\delta_a} & c_{11} \end{vmatrix}}{\Delta}$$

Viraje Estacionario - V

- A standard holding pattern uses right-hand turns and takes approximately 4 minutes to complete:
 - one minute for each 180 degree turn,
 - and two one-minute straight ahead sections).
 - Deviations from this pattern can happen if long delays are expected; longer legs (usually two or three minutes) may be used, or aircraft with distance measuring equipment (DME) may be assigned patterns with legs defined in nautical miles rather than minutes.
 - Less frequent turns are more comfortable for passengers and crew. Additionally, left-hand turns may be assigned to some holding patterns if there are airspace or traffic restrictions nearby.
- Aircraft flying in circles is an inefficient (and hence costly) usage of time and fuel, so measures are taken to limit the amount of holding necessary.
- Many aircraft have a specific *holding speed* published by the manufacturer; this is a lower speed at which the aircraft uses less fuel per hour than normal cruise speeds. A typical holding speed for transport category aircraft is 210 to 265 knots (491 km/h).

Viraje Estacionario – VI (Speed Limits)

- Speed Limits
 - Maximum holding speeds are established to keep aircraft within the protected holding area during their one-minute (one-minute and a half above 14,000 ft MSL – Mean Sea Level) inbound and outbound legs.
 - For civil aircraft (not military) in the United States, these airspeeds are:
 - Up to 6,000 ft MSL: 200 KIAS
 - From 6,001 to 14,000 ft MSL: 230 KIAS
 - 14,001 ft MSL and above: 265 KIAS
 - The ICAO Maximum holding speeds:
 - Up to 14000 ft: 230kts
 - 14000 ft to 20000 ft: 240kts
 - 20000 ft to 34000 ft: 265kts
 - Above 34000 ft: M0.83
 - With their higher performance characteristics, military aircraft have higher holding speed limits.

Viraje Estacionario – VII (Speed Limits)

- Speed Limits (cont)
 - In Canada the speeds are:
 - All propeller including turboprop aircraft :
 - Minimum Holding Altitude (MHA) to 30,000 ft (9,100 m): 175 kn (324 km/h; 201 mph)
 - Civilian Jet
 - MHA to 14,000 ft (4,300 m): 230 kn (426 km/h; 265 mph)
 - Above 14000 ft: 265 kn (491 km/h; 305 mph)
 - Climbing during the hold:turboprop - normal climb speed
 - Jet aircraft - 310 kn (574 km/h; 357 mph) maximum

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